Self-Sustaining Public Employment

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Abstract: In this paper we propose and consider a public employment program that accepts all applicants, pays each worker according to the value he adds to production, and aims to maximize worker surplus subject to a non-negative profit constraint. We construct an original model of production and compensation in which production inputs are discrete and heterogeneous, the division of surplus depends on firm objectives, and imperfect information reduces efficiency by limiting the search horizons of economic actors. In the baseline version of the model, the public employment program increases efficiency. In some alternative versions, the impact on efficiency is ambiguous. We draw on these possible sources of inefficiency when making practical suggestions for the program's design.

§ 1. Introduction

In standard economic theory, the well-being of workers does not enter directly into a firm's objective function. Instead, a firm maximizes profit, pays profit to its owners, and pays all remaining inputs the minimum amount needed to obtain them.[1]

Under certain felicitous conditions, it is not important whether a firm is owned by its suppliers of labor, capital, or land. For example, given perfect competition, homogeneity of inputs and firms, continuity, constant returns to scale, etc., the payment to each input tends toward its marginal product, and economic profits tend toward zero.[2]

However, when such conditions are not satisfied, economic profits may be positive, and the reservation wages of inputs may be substantially below what they contribute to their respective firms. In this case it *is* important whether firms are structured to serve the interests of labor, capital, or land. That is, if most firms are principally directed by owners of capital and land,[3] workers will be systematically underpaid relative to their contributions. This may violate fairness norms and contribute to income inequality.

Similarly, if labor markets were efficient, all unemployment would be voluntary, on the part of people with some combination of low skill, high non-labor income, and a high preference for leisure. However, in reality we observe markets that are beset by imperfect information. For example, individual workers may not be hired because their productivity is underestimated, and viable business ideas may not receive adequate financing because their future revenue streams are underestimated.[4] On a macroeconomic level, low employment and low demand may be mutually reinforcing; that is, lacking coordination, hiring increases and demand increases may each wait for each other, even when a simultaneous increase in both would be Pareto improving.[5]

Given imperfect labor markets, it is to be expected that many unemployed people could have unrealized potential to do work that would produce more than enough value to compensate them for their loss of leisure. For such people to remain unemployed represents a loss of efficiency, attributable to labor market failure and government policies that disincentivize work.

These two concerns — underpayment of labor and involuntary unemployment — both point to a common idea: Current labor markets may not be serving workers as well as they could. This in turn suggests a role for alternative productive institutions that focus on providing the best possible opportunities for workers.

The goal of creating worker-centric productive institutions is a broad one, admitting many promising approaches. This paper focuses on one particular proposal: a public employment program with the following four features.

(1) All those who are willing and able to work are admitted into the program.

(2) The labor of participants is combined with non-labor inputs, such as capital and land, to produce goods (and services).

(3) The value of the produced goods, net of payments to non-labor inputs, is divided among program participants according to their contributions.

(4) The objective of the program is to maximize the surplus of participating workers. Therefore, non-labor inputs are paid only their estimated reservation wages.

This proposal has a place in the debate between private and state-run production systems. The intention of the program is to create a level playing field between the two, so that workers within a given country can move back and forth from one to the other at will, depending on their individual preferences. That is, it aims to resolve the debate by means of a voting-with-feet equilibrium,[6] in which the size of each sector is determined by its ability to provide advantages relative to the other.[7]

This proposal is closely related to two other ideas: employer of last resort (ELR) programs[8] and worker cooperatives. All three ideas have the aim of organizing productive activity specifically for the benefit of workers. Thus, all three aim to increase the well-being of workers not by restricting economic activity that would occur otherwise (for example, by restricting foreign trade, restricting immigration, or imposing a minimum wage), but rather by creating new economic activity with the aim of generating additional economic surplus.

The program proposed here is like an ELR program and unlike worker cooperatives in that it is set up under the auspices of the government, accepts all applicants, and thus aims to increase overall economic stability by reducing the uncertainty that workers face regarding future employment.

The program proposed here is like worker cooperatives and unlike an ELR program in that it focuses mainly on the production of private goods (though it may produce public goods as well, provided that some government agrees independently to purchase them), finances wages in whole or in part with the sale of these goods, and allows for greater heterogeneity in the wages of workers.

The remainder of the paper is organized as follows.

In Section 2, we develop the model, which does not follow directly from any single pre-existing source. The model contains a finite number of firms and a finite number of discrete input units, grouped into categories of labor, capital, and land. Firms and input units are

both heterogeneous. That is, each firm has a potentially unique production function, which depends on the particular set of input units it employs. Also, each worker has utility that depends on his compensation and the particular firm he works for. No specific forms are assumed for the production functions; in this way, the model is highly general.

Wages are determined in part by firm objectives: Private firms maximize payments to their 'insiders', and pay only reservation wages to their 'outsiders'. The 'public firms' introduced by this proposal would maximize payments to labor, and pay reservation wages to capital and land.

To represent in simplified form the kind of imperfect information that drives labor market failure, we assume that only a small subset of all possible mappings of input units to firms are actually considered.

In Sections 3–5, we derive the primary results. In Section 3 we establish basic conditions under which individual input units are efficiently allocated within the bounds of the information constraint; this serves as a useful point of comparison in later analysis. In Section 4 we consider subsidies and taxes, thus exploring different budget constraints that the program might face. We show that efficiency is highest when subsidies are equal across sectors. For example, when a worker moves from unemployment to the public employment program, he should ideally continue to receive his prior social assistance payments *in addition to* the value he generates through work in the program. We also consider supplementing working income with lump-sum grants financed by a land tax, which in principle can improve the well-being of the poorest individuals without reducing efficiency.[9]

In Section 5 we show that the program leads to an overall increase in efficiency under specified conditions, such as equal subsidies across sectors. We follow this with an analysis of who is made better off and who is made worse off by the existence of the program.

In Sections 6–8, we broaden the discussion. In Section 6, we add four extra features to the model in turn. These are (1) an administrative cost variable, (2) uncertainty about the subsidies that program participants would hypothetically receive in the private sector, (3) uncertainty about the precise contributions of individual input units, and (4) uncertainty about the future revenue from current production, and thus about which investments by public firms will generate positive surplus. With each modeling change, we reconsider the result that the program leads to an increase in efficiency. In Section 7, we review concerns with and goals for the program, and use these to motivate preliminary suggestions for its design. In Section 8, we conclude.

§ 2. Model

In this model, the output value of each firm (whether public or private) depends on which input units it employs. These input units are discrete, heterogeneous, and are divided into the three categories of labor, capital, and land. Payments to inputs come from output value; the payment to each unit depends on the unit's replacement cost, its reservation wage, and

whether the welfare of the unit's owner is part of the firm's objective function. Public firms differ from private firms in that their main objective is to maximize the surplus of participating workers. Imperfect information limits the range of input-firm assignments that are considered.

§ 2.1. Production

Let l, k, and t be indexes for a unit of labor, capital, or land, respectively. Let L, K, and T be the sets of labor, capital, and land units. Let z be a generic index for an input unit of any type, and let Z be the set of all input units. Let i and j be firm indexes, and let I be the set of firms. Thus, there are $|Z| = |L| + |K| + |T|$ distinct input units, each of which can be employed, at a given time, by at most one of the $|I|$ distinct firms.

Let x_i be the set of input units employed by firm i, and let $i \leftarrow z$ represent the index i of the firm that employs input unit z. Let $y_i(x_i)$ be the value of firm i's output if it employs the factor units in set x_i:

$$y_i = y_i(x_i)$$

We do not consider whether each firm is producing consumption goods or capital goods, private goods or public goods, etc. Similarly, we do not consider output prices or quantities independently. Rather we simply suppose that a particular firm i, employing a particular combination of inputs x_i, will produce an output with a particular value, $y_i(x_i)$.

Input units are allowed to be heterogeneous in their contribution. That is, $y_i(x_i)$ is determined not just by how many units of each type the firm employs, but rather by which *specific* units it employs, and furthermore, which specific *combinations* of units it employs. We do not make any assumptions about the form of the production function.

The three types of inputs are distinct from each other in the following way. Labor is distinct in that a worker's choice of occupation may depend in part on which jobs offer a more pleasant work experience; we develop this in Subsection 2.3 below. Land (a category that can be broadened to include many other natural resources) is distinct from capital in that its supply doesn't depend on human choices in a previous period. Most of our analysis will take place in a static, single-period setting, but this distinction will be important when we consider the limits of non-distorting taxation in Subsection 4.2 below.

To simplify, we suppose that a firm's output depends only on which discrete input units it employs; that is, units cannot be hired in different ways or in different degrees of intensity. For example, we suppose that a worker-firm assignment $l \in x_i$ answers all relevant questions about work hours, work effort, work enjoyment, etc.

§ 2.2. Wages and profit shares

Let w_l^i, w_k^i, and w_t^i represent wage payments from firm i to worker l, capital unit k, or land unit t. Let w_z^i represent wage payments from firm i to a generic input unit z.

Firm i's profit Π_i is defined as the value of its output, minus the sum of the wages it pays to input units: $\Pi_i = y_i(x_i) - \sum_{z \in x_i}(w_z)$.

Rather than defining a separate class of agents as firm owners, suppose that profits are divided up among input units, and thus reach the owners of said units. That is, $\Pi_i = \sum_z(\pi_z^i)$, where π_z^i is the amount of firm i's profit that goes to the owner of input unit z.

Thus, production value is divided between wage payments and profit shares: $y_i(x_i) = \sum_{z \in x_i}(w_z^i) + \sum_z(\pi_z^i)$. Note that we are leaving open here the possibility that a person may receive profit shares from a firm despite neither working for that firm, nor owning an input unit employed by that firm — at least in the present period. In the base model we take this one step further and assume explicitly that profit shares are exogenous, or salable. We consider this assumption in more depth in a discussion following Proposition 1, in Section 3.

§ 2.3. Income and utility

We have already defined the components of income received by input units, i.e. wage payments and profit shares. We can thus write the income of input unit z employed by firm i as $w_z^i + \sum_j(\pi_z^j)$.

We have also specified that labor is distinguished by being the type of input that experiences differential utility depending on where it is employed. We formalize this idea with the equation $u_l^i = w_z^i + \sum_j(\pi_z^j) + v_l^i$, where we define v_l^i as the direct utility (or disutility if negative, but we make no assumptions about this either way) that worker l gains from employment at firm i, and u_l^i is the overall utility of the worker, which he endeavors to maximize.

We are assuming that work utility v_l^i can be expressed in terms of the same units as income. That is, the difference in utility between two jobs, $v_l^i - v_l^j$, is defined as the amount that worker l would be willing to pay to be employed at i rather than j, all else equal. Thus, the additive separability assumed in the form of the utility function implies that this willingness to pay does not vary with the worker's other income; in other words, we assume for simplicity that labor supply decisions are not influenced by wealth effects.

Note that job enjoyment v_l^i is allowed to be heterogeneous with respect to worker-firm combinations. For example, one worker might strongly prefer employment at firm i to employment at firm j at equal wages, while another worker prefers the opposite. This is a realistic feature of the model, capturing for example the fact that workers have different location preferences, different preferences regarding the type of work they do, etc.

In other words, we simply assume that each work utility value v_l^i is taken from an exogenous, $|L|$ by $|I|$ matrix of real numbers. We make no further assumptions, e.g. about whether the entries are positive or negative, correlated or uncorrelated, etc.

The objective of capital and land owners is a simplified version of the workers' objective: rather than maximizing the sum of income and work enjoyment, they simply aim to

maximize income. So, to economize on notation we can say that owners of inputs of all types seek to maximize the objective function

$$u_z^i = w_z^i + v_z^i + \sum_j(\pi_z^j)$$

while specifying that $v_z^i = 0, \forall i, \forall z \in (K \cup T)$.

§ 2.4. Replacement cost and contribution

To develop a theory of wage determination within this model, we first develop a measure of how much an input unit z contributes to firm i's production. Specifically, we define this in terms of how much it costs firm i if z is removed from its set of inputs x_i, assuming that i has the option to hire replacements for z. We refer to this as the replacement cost of input unit z at firm i, or the contribution of z to i, and denote it as Δ_z^i.

To find the formula for Δ_z^i, we consider first the direct reduction in output caused by the removal of unit z: we can write this as $y_i(x_i) - y_i(x_i \backslash \{z\})$.

Next, consider the cost of hiring input units to replace z so that output returns at least to its previous level. We can write this as $\min_{\vec{z}' \not\ni z} \left(\sum_{z' \in \vec{z}'} (\underline{w}_{z'}^i) : y_i(x_i \backslash \{z\} \cup \vec{z}') \geq y_i(x_i) \right)$. Here, $\vec{z}'$ is a set of input units that i might use to replace z (so by definition, $z \notin \vec{z}'$), and $\underline{w}_{z'}^i$ is the reservation wage of an input unit $z' \in \vec{z}'$, i.e. the lowest wage at which z' will accept employment at i. We also allow $\vec{z}'$ to contain elements that indicate the removal of other input units. For example, let $\vec{z}' = \{a, -b\}$ indicate hiring unit a, but firing unit b, so that $\sum_{z' \in \vec{z}'} (\underline{w}_{z'}^i) = \underline{w}_a^i - \underline{w}_b^i$.

The formula for the replacement cost Δ_z^i combines both of these ideas: The cost to firm i of unit z's departure may be a mix of lost output revenue and expenditure on replacements; the firm will therefore choose a set of replacement inputs to minimize the sum of these two types of loss, resulting in a replacement cost of

$$\Delta_z^i = \min_{\vec{z}' \not\ni z} \{ \ [y_i(x_i) - y_i(x_i \backslash \{z\} \cup \vec{z}')] \ + \ [\sum_{z' \in \vec{z}'} (\underline{w}_{z'}^i)] \ \}$$

So far we have defined Δ_z^i in terms of the cost to firm i of losing an input unit z that it already employs, given the opportunity to hire (and possibly fire) other units, i.e. those in $\vec{z}'$. We can also create a parallel definition in terms of the benefit to firm i of gaining an input unit z that it does not yet employ, given the opportunity to fire (and possibly hire) other units:

$$\Delta_z^i = \max_{\vec{z}' \not\ni z} \{ \ [y_i(x_i \cup \{z\} \backslash \vec{z}') - y_i(x_i)] \ + \ [\sum_{z' \in \vec{z}'} (\underline{w}_{z'}^i)] \ \}$$

These mirror-image definitions are equivalent, as long as we assume that the firm has settled all mutually beneficial transactions with other input units. In this case, we can use revealed preference to show that the set of newly-hired inputs $\vec{z}'$ which minimizes i's replacement cost if z quits is identical to the set of newly-fired inputs $\vec{z}'$ which maximizes i's gain if z begins

employment there. Therefore, under equilibrium conditions we can use the terms 'replacement cost' or 'contribution' interchangeably when referring to the variable Δ_z^i.

In a more standard production model, the closest analog to Δ_z^i is the marginal revenue product. Δ_z^i serves similar functions, such as wage determination, but it has been adapted to our present environment with discrete, heterogeneous input units.

§ 2.5. Hiring and payment

Recall that $\underline{w_z^i}$ is defined as unit z's reservation wage for working at firm i. The equation $w_z^i + v_z^i = w_z^j + v_z^j$ indicates that z is just indifferent between working at firm i and some other firm j. Therefore, rearranging the terms and using z's next-best alternative as the point of comparison, we can write $\underline{w_z^i} = \left[\max_{j \neq i}\left(w_z^j + v_z^j\right)\right] - v_z^i + \varepsilon$, where ε is an arbitrarily small positive number that will often be suppressed in the interest of visual clarity.

We assume that firm i will be willing to hire unit z if the wage needed to do so is below z's contribution Δ_z^i; this is intuitive, as i can clearly gain from the transaction . Similarly, we know by definition that z will accept employment at i if the offered wage exceeds z's reservation wage w_z^i. Putting these two ideas together, we hold that if unit z's contribution Δ_z^i exceeds its reservation wage w_z^i, z will be employed at i, with a wage $w_z^i \geq \underline{w_z^i}$.

We have not yet ruled out that z might be paid a wage in excess of its contribution, $w_z^i > \Delta_z^i$. Clearly, this is a losing transaction from the perspective of the rest of firm i, and thus it only makes sense if improving the welfare of unit z is a part of firm i's objective. Therefore, if i pays z more than Δ_z^i, we count the wage as $w_z^i = \Delta_z^i$, and count the remaining portion of the payment as a profit share, π_z^i.

We can summarize the last three paragraphs with the dual statement

$$\Delta_z^i > \underline{w_z^i} \Rightarrow \begin{cases} z \in x_i \\ w_z^i \in \left[\underline{w_z^i}, \Delta_z^i\right] \end{cases}$$

This says that if unit z's contribution at firm i is greater than its reservation wage, i will employ z, and pay a wage between the two values.

A consequence of this framework is that the wage w_z^i of unit z at firm i can be bid upward by an offer from another firm j; even if z remains employed at i, the offer from j can increase z's reservation wage $\underline{w_z^i}$, and thus z's actual wage $w_z^i \in \left[\underline{w_z^i}, \Delta_z^i\right]$.

We assume that a firm that cannot make non-negative profit while producing will shut down. In other words, if for any feasible input set x_i, the sum of reservation wages is greater than the value of output, the firm will neither hire nor produce. That is,

$$\max_{x_i \neq \emptyset}\left[y_i(x_i) - \textstyle\sum_{z \in x_i}\left(\underline{w_z^i}\right)\right] < 0 \Rightarrow \begin{cases} x_i = \emptyset \\ y_i(x_i) = y_i(\emptyset) = 0 \end{cases}$$

Unlike models in which wages are determined mechanically by the marginal revenue products of inputs, we see that an employment contract between z and i may generate some surplus equal to $\Delta_z^i - \underline{w}_z^i$, which leaves open the question of how that surplus will be divided. We address this next, in the context of describing the different objectives of private and public firms.

§ 2.6. Private firms and public firms

In this model, an individual firm i may be either a private firm, which we denote as $i \in A$, meaning that firm i is in 'Sector A' (the original factor market), or it may be a public firm, which we denote as $i \in B$, meaning that firm i is in 'Sector B' (the public employment sector).

A public firm is a productive entity created by the public employment program. Like a private firm, we assume that a public firm $i \in B$ produces value $y_i(x_i)$ based on the set of input units it employs, pays wages $w_z^i \in [\underline{w}_z^i, \Delta_z^i]$, and distributes the remaining revenue $\Pi_i = y_i - \sum_{z \in x_i}(w_z^i)$ in the form of profit shares π_z^i. Or it shuts down, if it is incapable of making non-negative profit.

The differences between private and public firms lie in their objectives, i.e. the constituency that they primarily hope to serve. We assume that every firm i has a set of 'insider' input units M_i whose welfare is part of its objective function, and a set of 'outsider' input units $\mathrm{N}_i = Z \backslash \mathrm{M}_i$ whose welfare is not a part of its objective function.

We assume that a member of firm i's insider set $z \in \mathrm{M}_i$ is paid a wage equal to its contribution, $w_z^i = \Delta_z^i$,[10] and that it may receive a positive share of the profit $\Pi_i = y_i(x_i) - \sum_{z \in x_i}(w_z^i)$. In contrast, a member of i's outsider set $z \in \mathrm{N}_i$ is paid only its reservation wage, $w_z^i = \underline{w}_z^i$, and receives no profit share.

We assume that the primary purpose of a public firm $i \in B$ is to maximize the surplus of participating workers. Therefore its insider set is $\mathrm{M}_i = L$.

We make no particular assumptions about the insider and outsider sets of private firms in our model, but intuitively one may imagine that the insiders are often capital and land owners (i.e. shareholders), company founders, and top management. When the decision making of a private firm is primarily directed by the owners of its capital or land, labor may receive a disproportionately small share of the surplus generated by the firm, i.e. a disproportionately small share of $y_i(x_i) - \sum_{z \in x_i}(\underline{w}_z^i)$.

In summary, every firm i offers wages as follows:

$$w_z^i = \begin{cases} \Delta_z^i & \text{if} \quad z \in \mathrm{M}_i \\ \min\{\underline{w}_z^i, \Delta_z^i\} & \text{if} \quad z \in \mathrm{N}_i \end{cases}$$

So far we have not considered unemployment, which is an important motivation for the public employment program. We denote that input unit z is unemployed by writing $z \in x_0$.

That is, they belong to the 'null firm', which is identified by the zero index $i = 0$, and characterized by zero production: $y_0(x_0) = 0$.

We consider the unemployed input units to be in the private as opposed to the public employment sector; that is, $0 \in A$. The rationale for this is that every input not specifically in the public employment program is by default in the private employment sector, i.e. Sector A, which comprises the factor market status quo prior to the creation of Sector B.

§ 2.7. Imperfect information

Define ξ as the complete list of $z \in x_i$ assignments. That is, $\xi = (x_0, x_1, \dots, x_{|I|-1})$ consists of the $|I|$ distinct input sets for the $|I|$ firms.

Define Ξ as the set of all possible ξs. Since there are $|I|^{|Z|}$ ways to divide $|Z|$ input units among $|I|$ firms, Ξ contains $|I|^{|Z|}$ possible ξs, which in turn each describe $|I|$ input sets.

It is unrealistic to expect that all of the $|I|^{|Z|}$ possible ξs in Ξ will be considered. For example, an individual worker will not consider employment at every single firm, and an individual firm will not consider every single worker; it is possible that worker l could be happier and more productive at firm j than at his current job with firm i, but that l is not hired by j simply because neither l nor j is aware of the other. Further, for any arbitrary set of input units $\vec{z}$, it is possible that a simultaneous reallocation of all units in $\vec{z}$ to form a new firm or expand an existing firm could increase the sum of production and utility $\sum_i(y_i(x_i)) + \sum_l(v_l^{i \leftarrow l})$, but that this doesn't occur because no one considers it.

A major share of factor market dysfunction can be explained in terms of this kind of imperfect information. With perfect information, each factor unit z would be able to choose from a large menu of $|I|$ employment opportunities, with each firm i offering a wage w_z^i that could be bid up to z's contribution Δ_z^i by competing offers. Coordinated reallocation of inputs could realize additional potential for mutual gains, thus minimizing involuntary unemployment.

Imperfect information is a complex and subtle phenomenon, but for present purposes we represent it in a relatively simple manner by supposing that some ξs are considered and thus in the realm of possibility, while others are not. That is, we define $\Omega \subsetneq \Xi$ as the set of $z \in x_i$ assignments ξ that are considered, and assume that an assignment list ξ that is not an element of Ω cannot occur.

Thus, we modify the search problem of input unit z from $\max_i(u_z^i)$ to $\max_{i:\xi \in \Omega}(u_z^i)$. Further, z's reservation wage is defined only with respect to other opportunities allowed by the information constraint, i.e. as $\underline{w_z^i} = \left[\max_{j \neq i:\xi \in \Omega}(w_z^j + v_z^j)\right] - v_z^i$.

The effect of the information constraint on individual input units and firms may be heterogeneous. Some workers may still have a great many employment opportunities to choose from, while others have very few, or even none. For example, an ex-convict may be

released from prison to find that his range of legal employment opportunities is severely restricted, because firms consider it risky to employ him. Similarly, capital owners, land owners, and firms might be affected by the information constraint in varying degrees.

§ 2.8. Efficiency

Because we have already defined the utilities from work experience v_l^i in the same units as the value of production output y_i, and we have assumed that the v_l^is are independent of worker income, we can measure efficiency simply in terms of the global sum of the two types of value, i.e. $\sum_i[y_i(x_i)] + \sum_l[v_l^{i\leftarrow l}]$. That is, if an assignment list ξ' produces a higher value of this sum than another assignment list ξ, it would be possible to Pareto-improve by changing assignments from ξ to ξ' and making the necessary side payments.

If the factor market were run by an omniscient social planner who sought to maximize the sum of production value and utility from employment and thus achieve full Pareto efficiency, his problem could be written as $\max_{\xi\in\Xi} \sum_i(y_i(x_i)) + \sum_l(v_l^{i\leftarrow l})$. Because of imperfect information, represented by a deep reduction in the set of available ξs, this maximum is unlikely to be feasible. However, we can proceed by comparing the efficiency of different situations relative to one another. Of particular interest is the question of whether overall efficiency is greater before or after the public employment program is introduced; we address this question using the core model in Section 5, then under various alternative assumptions in Section 6.

§ 2.9. Equilibrium

The model is in equilibrium when the following two statements hold for all input units z and all profitable firms i: $y_i(x_i) \geq \sum_{z\in x_i}(\underline{w}_z^i)$:

$$w_z^i = \begin{cases} \Delta_z^i & \text{if} \quad z \in \mathrm{M}_i \\ \min\{\underline{w}_z^i, \Delta_z^i\} & \text{if} \quad z \in \mathrm{N}_i \end{cases} \qquad\qquad i \leftarrow z = \underset{j:\, \xi\in\Omega}{\operatorname{argmax}}\left(w_z^j + v_z^j\right)$$

These two statements represent optimization on the firm side and on the input side, respectively. That is, each firm i offers wages equal to contributions Δ_z^i for insiders $z \in \mathrm{M}_i$, and it offers reservation wages to outsider units $z \in \mathrm{N}_i$ as long as those reservation wages are exceeded by the units' contributions. Given these offers, each input unit z chooses the firm that offers the best combination of wage compensation w_z^i and (in the case of workers) the direct utility from working v_z^j.

§ 3. Efficiency in the base model

In this section, we establish that in a baseline version of the model — which has no taxes, subsidies, or other complications — each individual input unit is employed as efficiently as it

can be while subject to the confines of the information constraint. We can formalize this condition as 'local efficiency', i.e. a situation in which

$$z \in x_i \Leftrightarrow i = \underset{j:\xi\in\Omega}{\operatorname{argmax}}\left(\Delta_z^j + v_z^j\right)$$

Proposition 1: In the base model, equilibrium implies local efficiency.

We want to show: $j \leftarrow z = \underset{j:\,\xi\in\Omega}{\operatorname{argmax}}\left(\Delta_z^j + v_z^j\right)$

(1.1) Define $\tilde{\iota_z} \equiv \underset{j:\,\xi\in\Omega}{\operatorname{argmax}}\left(\Delta_z^j + v_z^j\right)$.

(1.2) $\Delta_z^{\tilde{\iota_z}} + v_z^{\tilde{\iota_z}} > \Delta_z^j + v_z^j, \forall j \neq \tilde{\iota_z}: \xi \in \Omega$.

(1.3) Axiom from § 2.6: $w_z^i = \begin{cases} \Delta_z^i & \text{if} \quad z \in \mathrm{M}_i \\ \min\left\{\underline{w_z^i}, \Delta_z^i\right\} & \text{if} \quad z \in \mathrm{N}_i \end{cases}, \forall i, z.$

(1.4) $\Delta_z^i \geq w_z^i, \forall i, z$.

(1.5) From (1.2) and (1.4), $\Delta_z^{\tilde{\iota_z}} + v_z^{\tilde{\iota_z}} > w_z^j + v_z^j, \forall j \neq \tilde{\iota_z}: \xi \in \Omega$.

(1.6) Derivation from § 2.5 with constraint from § 2.7: $\underline{w_z^i} = \left[\max_{j\neq i:\xi\in\Omega}\left(w_z^j + v_z^j\right)\right] - v_z^i, \forall i, z$.

(1.7) $\underline{w_z}^{\tilde{\iota_z}} + v_z^{\tilde{\iota_z}} > w_z^j + v_z^j, \forall j \neq \tilde{\iota_z}: \xi \in \Omega$.

(1.8) From (1.3), $w_z^{\tilde{\iota_z}} \geq \min\left\{\underline{w_z}^{\tilde{\iota_z}}, \Delta_z^{\tilde{\iota_z}}\right\}$.

(1.9) From (1.5), (1.7), and (1.8), $w_z^{\tilde{\iota_z}} + v_z^{\tilde{\iota_z}} \geq \min\left\{\underline{w_z}^{\tilde{\iota_z}}, \Delta_z^{\tilde{\iota_z}}\right\} + v_{\tilde{\iota_z}} > w_z^j + v_z^j, \forall j \neq \tilde{\iota_z}: \xi \in \Omega$.

(1.10) $\underset{j:\,\xi\in\Omega}{\operatorname{argmax}}\left(\Delta_z^j + v_z^j\right) = \underset{j:\,\xi\in\Omega}{\operatorname{argmax}}\left(w_z^j + v_z^j\right)$. □

The intuition behind this proof is that, absent any specific distortions, an individual input unit's wage w_z^i in this model is sensitive enough to the input's replacement cost Δ_z^i to ensure that the unit will not be misallocated, at least within the limited field allowed by the information constraint $\xi \in \Omega \subsetneq \Xi$.

With a few more assumptions it should be possible to extend the result to a stronger efficiency criterion which requires that multiple factors moving simultaneously within the bounds of the information constraint can't achieve mutual gains in utility u_z^i. However, this added technical work would not substantially improve our ability to evaluate the public employment program, so we stick with the local efficiency criterion for simplicity.

In this proof we made use of the assumption that payments in excess of replacement costs would be considered as profit shares, and that profit shares are exogenous, or salable. For example, suppose that a person owns capital unit k which is employed at firm i, plus an ownership stake in i which comes with profit share π_k^i. What our assumption says is that if

the person chooses to reallocate unit k to another firm j, although he will no longer receive the wage payment w_k^i, he will still receive the profit share π_k^i. Or equivalently, that he can sell his profit share to another investor in firm i, at full value.

On the other hand, if firm i had a rule that its owners could not reallocate their productive assets to other firms without also forfeiting their profit share, this could cause frictions preventing the assets from being put to their best possible use. Firm owners seeking to avoid these frictions and the resulting inability to maximize the productivity of their assets should allow owners of inputs being reallocated elsewhere (whether labor, capital, or land) to retain or sell any associated profit shares, as this model assumes. But this may not always be appropriate in a public employment setting, where newly-joining workers may make a legitimate claim to a share of public sector profits. Therefore we return to this topic when discussing program design in Section 7.

§ 4. Subsidies and taxes

So far we have analyzed the public employment program as a series of public firms $i \in B$ that attempt to maximize the utility of participating workers by engaging in profitable production and giving as much as possible of the resulting surplus to labor. That is, a public firm i aims to pay each participating worker l his replacement cost Δ_l^i, plus a share π_l^i of the profit $\Pi_i = y_i(x_i) - \left[\sum_{l \in x_i}(\Delta_l^i) + \sum_{k \in x_i}(\underline{w}_k^i) + \sum_{t \in x_i}(\underline{w}_t^i)\right]$.

We cannot say in general terms what standard of living this will provide, or whether it is 'enough' to satisfy specified social welfare norms. Also, whereas Section 1 considers a simplified environment with no distorting complications, we should generalize to accommodate situations in which the private employment market (Sector A, which exists prior to the creation of the public employment program) already contains distorting subsidies.

For these two reasons, i.e. because they might both improve the welfare of low-income workers and establish a balance with existing subsidies, we consider providing subsidies to public employment program participants. Since we've assumed that firms cannot make payment in excess of revenues (whether public or private), asking what subsidies the program should provide is similar to asking what cost constraint it will face — a strict break-even requirement in the case of zero subsidies, or an accepted amount of net spending per participant in the case of positive subsidies.

§ 4.1. Balancing subsidies across occupations and sectors

In this subsection, we first show that if employment-specific subsidies change the allocation of input units, deadweight loss results. Second, we show that if a worker l receives the same subsidy in Sector B as he does in Sector A, the resulting foot-voting equilibrium is efficient.

We begin with some definitions. Let σ_z^i represent the subsidy that input unit z receives from the government if it is employed by firm i. Let $\tilde{\imath}_z = \underset{i}{\operatorname{argmax}}\left(\Delta_z^i + v_z^i\right)$ be the firm that would employ in the absence of subsidies. Let $\tilde{\jmath}_z = \underset{j}{\operatorname{argmax}}\left(\Delta_z^j + v_z^j + \sigma_z^j\right)$ be the firm that would employ unit z in the presence of subsidies. Recall that we represent unemployment as 'employment by firm zero', so that it fits into this framework. Indeed, the subsidy σ_l^0 received by an unemployed worker $l \in x_0$ is of central importance here.

Note that these maximization problems still take place within the context of limited information, represented by the constraint $\xi \in \Omega \subsetneq \Xi$, but that we write them in simplified form as $\max_i(\cdot)$ rather than $\max_{i:\xi\in\Omega}(\cdot)$. We will continue to suppress this additional notation from now on to save space, while noting here that the constraint still applies. We will also assume for simplicity that the solutions to these maximization problems are single-valued, i.e. that $\nexists i, j, z: \Delta_z^i + v_z^i = \Delta_z^j + v_z^i$. Since each input unit z considers a finite number of firms, and since each $\Delta_z^i + v_z^i$ can in theory take on an infinite number of possible values, this assumption is not counter-intuitive in any important way.

Here we introduce the concept of 'first order deadweight loss', denoted by Ψ. First, we define $\psi_z = \left(\Delta_z^{\tilde{\imath}_z} + v_z^{\tilde{\imath}_z}\right) - \left(\Delta_z^{\tilde{\jmath}_z} + v_z^{\tilde{\jmath}_z}\right)$ as the reduction in value (in terms of both replacement cost Δ_z^i and employment utility v_z^i) associated with a reallocation of z from $\tilde{\imath}_z$ to $\tilde{\jmath}_z$ that comes as a direct result of the subsidy scheme. Then, we define $\Psi = \sum_z(\psi_z)$ as the sum over all input units of these reductions. We refer to this as first order deadweight loss because it only takes into account the input units that the subsidy directly causes to move, as opposed to input units that might be hired to take their place, etc. This measure has the advantage of simplicity.

Proposition 2: Employment-specific subsidies generate deadweight loss.

We want to show: $\exists z: \tilde{\imath}_z \neq \tilde{\jmath}_z \Rightarrow \Psi > 0$

(2.1) For $z: \tilde{\imath}_z \neq \tilde{\jmath}_z$, $\Delta_z^{\tilde{\imath}_z} + v_z^{\tilde{\imath}_z} = \max_i\left(\Delta_z^i + v_z^i\right)$, and $\Delta_z^{\tilde{\jmath}_z} + v_z^{\tilde{\jmath}_z} \neq \max_i\left(\Delta_z^i + v_z^i\right)$

(2.2) For $z: \tilde{\imath}_z \neq \tilde{\jmath}_z$, $\psi_z = \left(\Delta_z^{\tilde{\imath}_z} + v_z^{\tilde{\imath}_z}\right) - \left(\Delta_z^{\tilde{\jmath}_z} + v_z^{\tilde{\jmath}_z}\right) > 0$

(2.3) For $z: \tilde{\imath}_z = \tilde{\jmath}_z$, $\Delta_z^{\tilde{\imath}_z} + v_z^{\tilde{\imath}_z} = \Delta_z^{\tilde{\jmath}_z} + v_z^{\tilde{\jmath}_z}$, so $\psi_z = 0$.

(2.4) From (2.2) and (2.3), $\exists z: \tilde{\imath}_z \neq \tilde{\jmath}_z \Rightarrow \Psi = \sum_z(\psi_z) > 0$. □

Proposition 3: Foot voting between sectors is efficient with equal subsidies.

Definitions: $\alpha_l = \underset{j\in A}{\operatorname{argmax}}\left(w_l^j + v_l^j + \sigma_l^j\right)$. $\beta_l = \underset{j\in B}{\operatorname{argmax}}\left(w_l^j + v_l^j + \sigma_l^j\right)$.

We want to show: $\left(\sigma_l^{\alpha_l} = \sigma_l^{\beta_l}\right) \Rightarrow \begin{cases} (i \leftarrow l) \in A & \text{if} \quad \underset{j \in I}{\operatorname{argmax}}\left(\Delta_l^j + v_l^j\right) \in A \\ (i \leftarrow l) \in B & \text{if} \quad \underset{j \in I}{\operatorname{argmax}}\left(\Delta_l^j + v_l^j\right) \in B \end{cases}$

(3.1) Axiom from § 2.5: Firm i pays worker l up to Δ_l^i, given a sufficient competing offer.

(3.2) $\alpha_l = \underset{j \in A}{\operatorname{argmax}}\left(\Delta_l^j + v_l^j + \sigma_l^j\right). \quad \beta_l = \underset{j \in B}{\operatorname{argmax}}\left(\Delta_l^j + v_l^j + \sigma_l^j\right).$

(3.3) $(i \leftarrow l) \in \begin{cases} A & \text{if} \quad \max\limits_{j \in A}\left(\Delta_l^j + v_l^j\right) + \sigma_l^{\alpha_l} > \max\limits_{j \in B}\left(\Delta_l^j + v_l^j\right) + \sigma_l^{\beta_l} \\ B & \text{if} \quad \max\limits_{j \in A}\left(\Delta_l^j + v_l^j\right) + \sigma_l^{\alpha_l} < \max\limits_{j \in B}\left(\Delta_l^j + v_l^j\right) + \sigma_l^{\beta_l} \end{cases}$

(3.4) If $\sigma_l^{\alpha_l} = \sigma_l^{\beta_l}$, $(i \leftarrow l) \in \begin{cases} A & \text{if} \quad \underset{j \in I}{\operatorname{argmax}}\left(\Delta_l^j + v_l^j\right) \in A \\ B & \text{if} \quad \underset{j \in I}{\operatorname{argmax}}\left(\Delta_l^j + v_l^j\right) \in B \end{cases}$ □

Proposition 3 is a straightforward extension of Proposition 1, and an approximate converse of Proposition 2. That is, Proposition 1 demonstrates baseline efficiency, Proposition 2 demonstrates inefficiency with unequal subsidies, and Proposition 3 demonstrates efficiency with equal subsidies.

Of course, Proposition 3 can be broadened to cover all input units z rather than just workers l without any additional effort (i.e. just by changing the index); we focus on labor to emphasize the connection with the idea of a foot-voting equilibrium.

In particular, this result hints at an extremely broad idea: an efficient foot-voting equilibrium between capitalism and socialism. Or, put more circumspectly, an efficient foot-voting equilibrium between traditional labor markets centered on private production (in which owners of capital and land often hire labor, and hold primary decision-making power) and a government-supported alternative production sector that takes the well-being of workers as its main objective.

In this equilibrium, the relative sizes of the two sectors (A and B) are determined by the share of workers who find themselves to be happier (v_l^i) and more productive (Δ_l^i) in one sector than in the other. A person l works in the private sector if he finds that a private firm $i \in A$ offers the highest value of $\Delta_l^j + v_l^j$, and thus the highest value of $w_l^j + v_l^j$. Or, person l works in the public sector if a public firm $i \in B$ maximizes $\Delta_l^j + v_l^j$. Since workers have different preferences and skills, and the two sectors have different advantages and disadvantages, it is likely that neither sector would employ one hundred percent of the workers, but rather that the two sectors would maintain a dynamic balance, with a ratio of employees that would shift according to tastes, technology, the effectiveness of the public employment program's organization and administration, etc.

Note that this is quite different from deciding the relative scope of private and state-run forms of production via a political process. That is, it has little in common with the idea of a

socialist revolution that would seize private property and abolish private production by force. It is also entirely distinct from the idea of determining the size of a country's public sector by a legislative process, e.g. by majority voting among elected officials.

Instead, individual workers choose between the two sectors with minimal friction; rather than needing to migrate to a more socialist or a more capitalist jurisdiction to suit their preference, they simply enroll in or withdraw from the public employment program.

Thus, offering the same subsidies to a worker regardless of their choice of sector allows a level playing field for competition (and complementarity) between traditional capitalist labor markets — with their attendant levels of unemployment — and a worker-centric alternative. Of course, it is not always trivial to determine the precise subsidy $\sigma_l^{\alpha_l}$ that a worker l participating in Sector B would have received in Sector A, because the solution α_l to their conditional maximization problem $\max_{j \in A}\left(w_l^j + v_l^j + \sigma_l^j\right)$ is not observed. So in practice this is something to be approximated — a difficulty we will discuss in Subsection 6.2.

§ 4.2. Land value taxation

In this subsection, we continue to explore the limits of government payments to public employment participants that are consistent with efficiency. To that end, we consider a lump-sum subsidy that would come from a fund generated by a tax on land.

We define τ_z^i as the tax charged to input unit z if employed by firm i.

Generally speaking, taxes on capital and labor cause distortion and deadweight loss. In the case of capital, it is logical that higher capital taxes in the current period will reduce the amount of capital that will be produced for use in future periods. In the case of labor, reductions in work effort are a likely result of a tax that depends positively on income. These dynamics lie largely outside our model, but we mention them to explain the motivation for taxing land in particular.

On the other hand, land is by construction the type of production input that is fundamentally exogenous to human effort (and thus we may define the category broadly to include other types of natural resources that are both scarce and valuable), which means that it can be taxed without distortion, unless (a) the tax creates a bias toward one use of land over another or (b) the tax actually exceeds the value of the land, prompting the owner to abandon it. If these two conditions are absent, i.e. if the tax on each land unit τ_t^i is independent of i and if $\tau_t < \max_i\left(\Delta_t^i\right)$, revenue can be raised without distortion.[11] Then, if the revenue is distributed lump sum, a complete cycle of redistribution can be accomplished without a loss of efficiency. We formalize this in the following proposition.

Proposition 4: A land tax returned lump sum is non-distorting.

We want to show: $\left.\begin{matrix} \tau_t^i = \tau_t\,, \forall i, \forall t \\ \sigma_l^i = \sigma_l, \forall i, \forall l \\ \tau_t < \max_i\left(\Delta_t^i\right), \forall t \end{matrix}\right\} \Longrightarrow (i \leftarrow z) = \underset{j}{\text{argmax}}\left(\Delta_z^j + v_z^j\right), \forall z.$

(4.1) By definition of land (versus capital), current land profits don't affect future land supply.

(4.2) Because $\tau_t < \max_i \Delta_t^i$, t can obtain $w_t^i > \tau_t$, and therefore does not abandon the asset.

(4.3) By construction, τ_t is independent of $i \leftarrow t$.

(4.4) From (4.2) and (4.3), $\underset{i}{\text{argmax}}\left(w_t^j - \tau_t\right) = \underset{i}{\text{argmax}}\left(w_t^j\right)$.

(4.5) By construction, σ_l is independent of $i \leftarrow l$.

(4.6) From (4.5), $\underset{j}{\text{argmax}}\left(w_l^j + \sigma_l + v_l^j\right) = \underset{i}{\text{argmax}}\left(w_l^j + v_l^j\right)$.

(4.7) From (4.1), (4.4), (4.6), and (1), $(i \leftarrow z) = \underset{j}{\text{argmax}}\left(\Delta_z^j + v_z^j\right), \forall z.$ □

An additional advantage of land value taxation is that it may satisfy fairness norms that are not satisfied by labor and capital taxation. That is, instead of taxing value that people can fairly claim they created themselves, it taxes value created by nature.

However, although this logic works well on an ongoing basis, landowners can argue at the moment of implementation that they paid for their land in good faith with income that did come from their own labor. Therefore, in the transition we should consider fully or partially compensating landowners for decreases in their net worth below a specified amount.

§ 4.3. Summary and additional discussion

We've assumed that the public employment program pays each participating worker l according to the value he adds to production of the public firm $i \in B$ that employs him, i.e. his replacement cost Δ_l^i plus a share of the profit $\Pi_i = y_i(x_i) - \sum_{k \in x_i}\left(\underline{w}_k^i\right) - \sum_{t \in x_i}\left(\underline{w}_t^i\right)$. Because some of the workers most in need of assistance will have low productivity and thus low Δ_l^is, we are interested in what other subsidies they might receive.

In Propositions 2-3, we found that if the public employment program pays worker l the same subsidy that he would receive if employed or unemployed in the private sector, this does not cause distortion; on the contrary, it removes a source of distortion that would exist otherwise. In Proposition 4, we found that if workers receive additional lump sum subsidies funded by a tax on land, no new distortion is introduced.

Of course, this in itself is not a policy prescription, but rather a simplified sketch of what policies are most consistent with efficiency. In addition to considering efficiency, policy

makers should consider equity, and consider how the measures described above may best be integrated with the existing structure of taxes and subsidies.

For example, by making the payments to workers from the land tax negatively dependent on income, we could gain equality at the expense of efficiency, which may be desirable up to a point. Similarly, although we have taken pre-existing worker subsidies in the private sector $\sigma_l^{i \in A}$ as given — of particular importance here are social insurance payments made to the unemployed (σ_l^0) and to the working poor — we should note that their existence suggests a source of distortion that the program does not directly resolve. We have established the broad principle that a participating low-income worker $l \in x_i$ should receive public employment payments $\Delta_l^i + \pi_l^i$ in addition to, rather than instead of his prior welfare benefits in the private sector, so that they are not driven toward suboptimal uses of their time, such as unemployment. But in trying to estimate and duplicate a participating worker's hypothetical private sector subsidy σ_l^A, we face similar information problems as those designing the initial subsidy scheme. However, if the program is successful at generating substantial increases in economic opportunity for previously low-income people, it would be possible to decrease these distorting subsidies while still enjoying a net increase in equality.

We have not considered taxes other than those on land except in passing, but it should be assumed that some distorting taxes exist to begin with, which finance public goods, the initial subsidies, etc. Thus, if we add a tax on land and return it lump sum as described above, we are accomplishing redistribution without introducing additional distortion, but we are not creating a state without distortion.

§ 5. Cost-benefit analysis

In this section we consider the program's benefits relative to its costs. First, we show that the total benefits are greater than the total costs when no distorting subsidies or other complications are present — a result we will question under varying specifications in Section 6. Second, we consider who is made better off and worse off by the program.

§ 5.1. Impact of the program on efficiency

In this analysis, we suppose that Sector A exists prior to the creation of Sector B, and that it is in equilibrium at time 0. Then, the public employment program launches, and the factor market evolves in a series of iterations $1, 2, \dots, N$ as input units move from firm to firm in response to changed conditions.

Let $z \to i \to j$ indicate that input unit z moves from firm i to firm j. Define ξ^0 as the assignment list in the initial equilibrium, ξ^N as the final equilibrium, and $\xi^0, \xi^1, \xi^2, \dots, \xi^N$ as the full sequence of assignment lists from beginning to end; every time a single input unit moves from one firm to another,[12] we progress from assignment list ξ^n to assignment list ξ^{n+1}.

Proposition 5: Absent distorting subsidies, the program increases efficiency.

We want to show:

$$\left.\begin{matrix} N > 0 \\ \sigma_z^i = \sigma_z^j, \forall i,j,z \end{matrix}\right\} \Rightarrow \sum_i [y_i(x_i : \xi^N)] + \sum_l \left[v_l^{i \leftarrow l : \xi^N}\right] > \sum_i [y_i(x_i : \xi^0)] + \sum_l \left[v_l^{i \leftarrow l : \xi^0}\right].$$

(5.1) Axiom from § 2.3: $[z \to i \to j] \Rightarrow \left[\Delta_z^j + v_z^j + \sigma_z^j > \Delta_z^i + v_z^i + \sigma_z^i\right]$

(5.2) By construction, $\sigma_z^i = \sigma_z^j, \forall i,j,z.$

(5.3) From (5.1) and (5.2), $[z \to i \to j] \Rightarrow \left[\Delta_z^j > \Delta_z^i + v_z^i - v_z^j\right]$

(5.4) $[z \to i \to j] \Rightarrow \left[\left(\Delta_z^j - \Delta_z^i\right) + \left(v_z^j - v_z^i\right) > 0\right]$

(5.5) $\sum_i [y_i(x_i : \xi^{n+1})] + \sum_l \left[v_l^{(i \leftarrow l) : \xi^{n+1}}\right] > \sum_i [y_i(x_i : \xi^n)] + \sum_l \left[v_l^{(i \leftarrow l) : \xi^n}\right].$

(5.6) By induction from (5.5), $\sum_i [y_i(x_i : \xi^N)] + \sum_l \left[v_l^{i \leftarrow l : \xi^N}\right] > \sum_i [y_i(x_i : \xi^0)] + \sum_l \left[v_l^{i \leftarrow l : \xi^0}\right].$ □

The intuition behind this proof is straightforward: Since input units are only reallocated when their contribution at the new firms (inclusive of worker utility) is greater than at the old firms, any reallocation that takes place after the program's inception increases economic surplus. The public program creates opportunities for worker-centered productive enterprise, which we can think of as expanding the set of firms I and the set Ω of possible input-firm configurations ξ. If any workers avail themselves of these opportunities (i.e. if $N > 0$, meaning that at least some reallocation takes place), their choices reveal that the new opportunities were superior to the old in terms of the sum of added production value and the experiential value of labor.

§ 5.2. Winners and losers

We have shown that in a simple version of the model with universally equal per-unit subsidies $\sigma_z^i = \sigma_z^j = \sigma_z$, the benefits of the program to aggregate value $\sum_i [y_i(x_i)] + \sum_l \left[v_l^{i \leftarrow l}\right]$ outweigh the costs. We can interpret the σ_zs broadly so that they can also represent taxes by taking on negative values, so that the scheme from Subsection 4.3 combining land value taxes with a lump sum transfer can be included in the result. In this subsection, we consider how those benefits and costs are distributed.

The pattern of gains and losses associated with the movement of an input unit z from one firm i to another firm j depends on whether z is an 'insider' or an 'outsider' at each of the two firms. We define these terms as follows: Recalling the assumptions about public and private firm objectives in Subsection 2.6, let an insider at a private firm $i \in A$ be a unit in the firm's ownership set, $z \in \mathrm{M}_i$, and let an outsider be a unit that is employed by the firm is not in its ownership set, $z \in (\mathrm{N}_i = x_i \backslash \mathrm{M}_i)$. Let an insider at a public firm $j \in B$ be a worker it employs, $z \in \left(L \cap x_j\right)$, and let an outsider be a unit of capital or land it employs, $z \in \left((K \cup T) \cap x_j\right)$.

Having defined these terms, we consider major gains and losses that result from the program's creation, in five parts.

First, consider the gains experienced by an input unit that moves from its initial employment at a private firm $i \in A$, to new employment at a public firm $j \in B$.

If l is a worker who starts as an insider $l \in \mathrm{M}_i$ in the private firm $i \in A$, and becomes an insider in the public firm $j \in B$, his wage plus work utility improves from $\Delta_l^i + v_l^i$ to $\Delta_l^j + v_l^j$. We call this a 'one-step improvement', because it corresponds to the added value from 'trading up' from one firm to the next.

If l is a worker who starts as an outsider $l \in \mathrm{N}_i$ in the private firm $i \in A$, his reservation wage before public employment is available is $\underline{w_l^i} = \max_{i' \in A} \left(\Delta_l^{i'} + v_l^{i'}\right) - v_l^i$, and thus his initial wage plus work utility is $w_l^i + v_l^i = \max_{i' \in (A \setminus \{i\})} \left(\Delta_l^{i'} + v_l^{i'}\right)$. If l moves to $j \in B$ and thus becomes an insider there, his wage plus work utility improves to $\Delta_l^j + v_l^j$. We call this a 'two-step improvement', because l improves from $\underline{w_l^i} + v_l^i$ to $\Delta_l^i + v_l^i$ and then again to $\Delta_l^j + v_l^j$.

If z is a capital or land unit that starts as an insider $z \in \mathrm{M}_i$ in the private firm $i \in A$, and becomes an outsider in the public firm $j \in B$, its gain will in theory be negligible. That is, $z \in \mathrm{M}_i$ is already paid its replacement cost $w_z^i = \Delta_z^i$ at the private firm, and the public firm only wants to pay the minimum amount needed to obtain it, which is its reservation wage $\underline{w_z^j} = w_z^i = \Delta_z^i$, or just marginally more. We call this a 'zero-step improvement'.

If z is a capital or land unit that starts off as an outsider $z \in \mathrm{N}_i$ in the private firm $i \in A$, and becomes an outsider in the public firm $j \in B$, its wage improves from $\underline{w_z^i} = \max_{i' \in A} \left(\Delta_z^{i'}\right)$ to approximately $\underline{w_z^j} = \Delta_z^i$. This is another one-step improvement.

Second, consider the impact of unit z's move from $i \in A$ to $j \in B$ on other units z'.

If z is initially an outsider $z \in \mathrm{N}_i$ in the private firm $i \in A$, with wage $w_z^i < \Delta_z^i$, z's departure will reduce i's profit, and make i's insiders $z' \in \mathrm{M}_i$ worse off. i's other outsiders $z' \in \mathrm{N}_i$ will not experience this effect. Meanwhile, if z is a capital or land unit, and thus becomes an outsider in the public firm, with wage $w_z^j < \Delta_z^j$, employees of the public firm $l \in x_j$ will be made better off.

Another unit z' might experience a second-order effect whereby the departure or arrival of z changes the replacement cost and thus the wage of z'. This change may be positive or negative; roughly speaking, it depends on whether z and z' are complements or substitutes. The movement of z from $i \in A$ to $j \in B$ can reduce the productivity of a complementary input $z' \in x_i$, but since this input can also be potentially reallocated to j, this effect may be mitigated. Meanwhile, substitute inputs $z' \in x_i$ are made better off.

Third, consider a previously unemployed input unit $z \in x_0$, which finds employment at a public firm $j \in B$. If this is a worker, his utility improves from $\sigma_l^0 + v_l^0$ to $\Delta_z^j + \sigma_l^0 + v_l^j$, provided that the public employment program is able to correctly match the subsidy σ_l^0. An unemployed capital or land unit improves only marginally from σ_z^0 (a zero-step improvement), but the resulting production increase in production value Δ_z^j benefits program participants $l \in x_j$.

Fourth, consider an input unit z that is initially employed by a private firm $i \in A$, and receives a competing offer from a public firm $j \in B$ that doesn't induce it to move, but does increase its reservation wage at i, $\underline{w}_z^i$. If z is an outsider ($z \in \mathrm{N}_i$), z will be made better off by the same amount that firm owners $z \in \mathrm{M}_i$ are made worse off. If z is an insider ($z \in \mathrm{M}_i$), no change will occur.

Fifth, if the program includes a land value tax system as described in Subsection 4.2, this makes landowners worse off by the same amount that it makes recipients of the resulting subsidy increases better off.

To summarize the above: By design, labor receives systematically higher gains from the public employment program than capital and land. Also, input units that are initially underpaid relative to their contribution stand to gain, while the owners of the firms that underpay them stand to lose. Unemployed units may gain an amount equal to the greatest contribution they can make in production. Depending on accepted fairness norms, one may argue that any or all of these represent increases in equity, which the program brings about while also increasing efficiency.

§ 6. Alternative assumptions

In this section, we consider four variations on the primary model: one in which an administrative cost variable is brought into consideration, and three in which some kind of uncertainty is added, viz. uncertainty about the value of hypothetical private sector subsidies, uncertainty about the contributions of individual input units, and uncertainty about the future return from current investments.[13]

§ 6.1. Administrative cost

In Proposition 5, we find that the public employment program increases efficiency, essentially because all resulting input unit reallocations are undertaken voluntarily by their owners. In this framework, the addition of public firms to the existing set of private firms allows for the creation of new surplus by introducing possible production relationships that were not available before. Such a benefit can be modeled as an expansion of Ω, the set of input-firm mappings ξ that fall into the realm of consideration. If the public employment program expands this set without cost, it is logical that it will increase efficiency. But what if there is a cost? We explore this question here.

We describe an add-on to the model, in which the public employment program expands Ω through the use of 'administrative units'. Let a be an index for an administrative unit, let c_a be the cost of employing this unit, let A be the set of administrative effort units that can possibly be employed, and let $\Phi \subset A$ be the set of administrative effort units that are actually employed.

We are interested first in the question of when the program should employ an additional set of administrative units, $\vec{a}$. Define Φ^0 and Φ' as the set of administrative units employed before and after employing $\vec{a}$. That is, $\Phi' = \Phi^0 \cup \vec{a}$, and so $\Omega(\Phi') \supset \Omega(\Phi^0)$.

Since the equilibrium assignment list ξ depends on the set of considered assignments Ω, which in turn depends on the administrative input set Φ, we use the notation $i \leftarrow z{:}\,\Phi$ to represent the equilibrium assignment of input unit z given Φ. We define $x_i{:}\,\Phi$ analogously.

When efficiency is the goal, the public employment program should not employ the administrative units in $\vec{a}$ if the surplus it adds in the equilibrium is outweighed by its cost. Call this the administrative cost rule. We can write this rule formally as

$$\sum_i\left(y_i(x_i{:}\,\Phi')\right) + \sum_l\left(v_l^{i\leftarrow l:\Phi'}\right) - \sum_{a\in\Phi'}(c_a) \geq \sum_i\left(y_i(x_i{:}\,\Phi^0)\right) + \sum_l\left(v_l^{i\leftarrow l:\Phi^0}\right) - \sum_{a\in\Phi^0}(c_a)$$

$$\sum_{a\in\vec{a}}(c_a) \leq \sum_i\left(y_i(x_i{:}\,\Phi')\right) - \sum_i\left(y_i(x_i{:}\,\Phi^0)\right) + \sum_l\left(v_l^{i\leftarrow l:\Phi'}\right) - \sum_l\left(v_l^{i\leftarrow l:\Phi^0}\right)$$

If the program is able to obey this rule, the result of Proposition 5 can be extended to the case with administrative cost. That is, if the rule is obeyed, it follows once again that any costs associated with the creation of the public employment must be outweighed by benefits. The logic is not substantially different: If each expenditure of administrative effort increases total surplus, and each voluntary reallocation of input units given a particular administrative infrastructure increases total surplus, the public employment program must increase total surplus overall.

However, there is no a priori certainty that a public employment program with non-zero size will be able to satisfy the administrative cost rule. Similarly, there is no a priori certainty that a program large enough to guarantee full employment will be able to satisfy the rule. Whether each of these is the case depends on how much administrative cost is needed to bring about the desired expansions of Ω. Nonetheless, although the program does not increase efficiency in these cases, arguments may still be made on the grounds of equity, or with a larger model, on the grounds of macroeconomic stability.

As a practical matter, the components of the administrative cost rule in its explicit form above are not easily estimated. Therefore, an easier way to implement the rule would be to include administrative cost in the program's budget, effectively treating it as another input type, along with non-administrative labor, capital, and land. Then, the rule is implemented by shutting down subunits of the program that cannot make non-negative profit after these administrative costs are taken into account.

§ 6.2. Uncertainty about subsidies

In Subsection 4.1, we showed that the division of inputs between the private and public employment sectors is efficient if the subsidy received by each worker is independent of which sector he is employed in. But we also acknowledged a natural problem: If worker l is currently participating in the public employment program, the subsidy that he *would have* received otherwise, $\sigma_l^{\alpha_l}$, is a counterfactual which may not be directly observable by public employment program administrators.

If the subsidy given to worker l in the public program is greater than the subsidy he would have received otherwise — i.e. if $\sigma_l^{\beta_l} > \sigma_l^{\alpha_l}$ — there is a bias pressing worker l toward public employment. Conversely, if $\sigma_l^{\beta_l} < \sigma_l^{\alpha_l}$, there is a bias pressing worker l away from public employment, and thus most likely toward unemployment, $\alpha_l = 0$.

In this environment, we revisit the result of Proposition 5, i.e. the result that the program increases efficiency overall. If a worker l is misallocated into Sector A because $\sigma_l^{\beta_l} < \sigma_l^{\alpha_l}$, this does not mean that the public employment program itself is causing a loss in efficiency; rather, it is merely failing to realize a potential gain in efficiency. However, if l is misallocated into Sector B because $\sigma_l^{\beta_l} > \sigma_l^{\alpha_l}$, we may say that the program is causing a loss in efficiency. This loss is equal to $\left(\Delta_l^{\alpha_l} + v_l^{\alpha_l}\right) - \left(\Delta_l^{\beta_l} + v_l^{\beta_l}\right)$, for a worker l such that

$$\left(\Delta_l^{\alpha_l} + v_l^{\alpha_l} > \Delta_l^{\beta_l} + v_l^{\beta_l}\right) \wedge \left(\Delta_l^{\alpha_l} + v_l^{\alpha_l} + \sigma_l^{\alpha_l} < \Delta_l^{\beta_l} + v_l^{\beta_l} + \sigma_l^{\beta_l}\right)$$

When considering the overall impact of the program on efficiency, these losses should be set against the gains in efficiency made when other input units are reallocated into the program. Such a gain is equal to $\left(\Delta_z^{\beta_z} + v_z^{\beta_z}\right) - \left(\Delta_z^{\alpha_z} + v_z^{\alpha_z}\right)$, for an input unit z such that

$$\left(\Delta_z^{\beta_z} + v_z^{\beta_z} > \Delta_z^{\alpha_z} + v_z^{\alpha_z}\right) \wedge \left(\Delta_z^{\beta_z} + v_z^{\beta_z} + \sigma_z^{\beta_z} > \Delta_z^{\alpha_z} + v_z^{\alpha_z} + \sigma_z^{\alpha_z}\right)$$

All else equal, the latter case would be more frequent than the former, because its two necessary inequalities point in the same direction rather than in opposite directions,[14] and because it includes capital and land units as well as labor units. But strictly speaking, the overall result that the gains from the public employment program outweigh the losses is not certain in this environment. If the public employment subsidies $\sigma_l^{\beta_l}$ are dramatically higher than the corresponding private sector subsidies $\sigma_l^{\alpha_l}$ for a large number of workers l, and if the surplus gains offered by the public employment program are extremely meager, the opposite result is possible.

However, we should recall that distorting subsidies create a problem that exists prior to the formation of a public employment program. For example, a worker l may be pressed toward unemployment by an unemployment subsidy. This results in an efficiency loss equal to $\left(\Delta_l^{\tilde{\iota}_l} + v_l^{\tilde{\iota}_l}\right) - \left(v_l^0\right)$, for l such that $v_l^0 + \sigma_l^0 > \Delta_l^{\tilde{\iota}_l} + v_l^{\tilde{\iota}_l} + \sigma_l^{\tilde{\iota}_l} > v_l^0$. If the public employment program is able to raise the income of poorer individuals through productive enterprise, it

may be possible and desirable to reduce some unemployment subsidies, and thus to enjoy a reduction in their attendant inefficiency without a loss in equity.

§ 6.3. Uncertainty about contributions

If a public firm $j \in B$ underestimates the contribution of input z, offering a wage $w_z^j < \underline{w_z^j} < \Delta_z^j$ which is insufficient to hire z away from its prior allocation at firm $i \in A$, an opportunity for a surplus gain of $\left(\Delta_z^j + v_z^j\right) - \left(\Delta_z^i + v_z^i\right)$ is not realized, but no surplus is lost.

On the other hand, it if public firm j overestimates unit z's contribution Δ_z^j, and offers a wage $w_z^j > \underline{w_z^j} > \Delta_z^j$, it is possible that the efficiency of z's allocation will be reduced as a consequence, by an amount equal to $\left(\Delta_z^i + v_z^i\right) - \left(\Delta_z^j + v_z^j\right)$. However, the condition for z's acceptance of j's offer, $w_z^j > \underline{w_z^j}$, and the composition of z's reservation wage, $\underline{w_z^j} = \Delta_z^i + v_z^i - v_z^j$, imply that such an efficiency loss cannot be greater than the amount of overpayment, i.e. that $\left(\Delta_z^i + v_z^i\right) - \left(\Delta_z^j + v_z^j\right) \leq w_z^j - \Delta_z^j$. Therefore, if j is held to a non-negative profit constraint as described in Subsection 2.5, the firm may only exist if all such accidental overpayments $w_z^j > \Delta_z^j$ are at least offset by the surplus generated by the firm, $y_j\left(x_j\right) - \sum_z\left(\underline{w_z^j}\right)$. Therefore, in this case we can rule out the possibility that the public firm j reduces efficiency overall.

One caveat should be considered, however: When the contribution values Δ_z^i are not transparent, there may be a transaction cost inherent in estimating or debating them. If this transaction cost is higher than usual within the public employment program, it could be a significant source of inefficiency.[15]

§ 6.4. Uncertainty about outputs

Suppose that public firm j has imperfect knowledge regarding what the output value $y_j\left(x_j\right)$ will be with a given set of inputs x_j, estimating it as $\hat{y}_j\left(x_j\right)$. If the firm proceeds on the basis of an estimate that is too high, $\hat{y}_j\left(x_j\right) > y_j\left(x_j\right)$, it is possible that although it expects to obey the non-negative profit constraint, it will fail to do so. That is, $\hat{y}_j\left(x_j\right) \geq \sum_{z \in x_j}\left(w_z^j\right)$, but $\Pi_j = y_j\left(x_j\right) - \sum_{z \in x_j}\left(w_z^j\right) < 0$. In some cases, the firm could even fail to generate sufficient value to cover its inputs' reservation wages, and thus produce negative surplus ex post. That is, $y_j\left(x_j\right) - \sum_{z \in x_j}\left(\underline{w_z^j}\right) < 0$.

In practice, failures of this kind are common in the private sector as well. That is, an investor believes that an enterprise will produce enough revenue to cover its costs, but this belief turns out to be mistaken, and the enterprise gives negative profit. However, the private market has an important feature which serves as a natural check against this: those who lose money as a result of unwise investments will have less money to invest thereafter. In contrast, one can imagine a government that continues indefinitely to create efficiency-

reducing enterprises, i.e. those with $y_j(x_j) < \sum_z(\underline{w}_z^j) < \hat{y}_j(x_j)$, while compelling taxpayers to bear the cost.

Thus, we have an important desideratum when it comes to the design of the public employment program: It should be structured so that poor investment decisions are kept in check. This isn't to say that poor investments can be eliminated altogether, but rather that they should be reduced to a rate comparable to the private sector, and that unexpected losses by some public firms should be approximately equal to unexpected gains by others.

But how may such desiderata be arranged for in practice? We turn to a preliminary discussion of this in the next section.

§ 7. Organizational structure

This section proceeds in three parts. First, we review desiderata that may be sought in the design of the public employment program. Second, we make practical suggestions for program design. Third, we discuss the extent to which the stated desiderata may be achieved by the stated program design suggestions.

§ 7.1. Desiderata

The following desiderata correspond to concerns with the program that have arisen earlier in our analysis. Per our discussion in Subsection 6.2, it is logical to seek a subsidy scheme that does not bias workers toward public employment as opposed to private employment when the latter is a more efficient allocation, or vice versa. Per our caveat in Subsection 6.3, it is logical to seek a system that determines wages (and by extension, other workplace decisions such as the division of tasks) without excessive transaction cost. Per our discussion in Subsection 6.4, it is logical to seek a means of preventing unexpected negative profits from systematically outweighing unexpected positive profits. Per our discussion in Section 3, we must consider how public firm profit shares should be divided.

In addition, we consider two desiderata arising from the program's initial motivation. First, it is intended as an environment in which the utility of workers is of primary importance. Second, in motivating the program we posited that many private firms are dominated by insiders who are insufficiently interested in providing beneficial opportunities to outsiders, who are therefore underpaid or unemployed. Given this motivation, it follows that the program should be structured to resist the formation of insider groups *within* it, who would pursue their own interests at the expense of broader gains.

§ 7.2. Program design

In this subsection, we give preliminary suggestions for the program's design. These suggestions are not intended to describe the program in every detail, but rather to provide

insight into how some of the organizational challenges involved in such an undertaking might be addressed.[16]

As in our formal model, the program is made up of several semi-autonomous public firms $i \in B$. Each of these can be shut down independently of the others if it is unable to generate positive surplus $y_i(x_i) - \sum_{z \in x_i}(\underline{w}_z^i) > 0$, and thus unable to satisfy the non-negative profit constraint $y_i(x_i) - \sum_{z \in x_i}(w_z^i) \geq 0$.

The authority (and possibly the material compensation) of administrators (or alternatively, administrative teams) depends on the success of public firms they have coordinated and approved funding for in the past. That is, administrators whose projects have generated a great deal of surplus $y_i(x_i) - \sum_{z \in x_i}(\underline{w}_z^i)$ will be given greater leeway to authorize investments with uncertain returns. Conversely, administrators whose projects have generated negative surplus $y_i(x_i) - \sum_{z \in x_i}(\underline{w}_z^i) < 0$, and thus negative profit $y_i(x_i) - \sum_{z \in x_i}(w_z^i) < 0$, will experience a reduction in authority, and in some cases lose their positions entirely. Further, administrators may acquire merit through other measures of performance, such as providing opportunities in particular to those members of society in greatest need of assistance.

The program has a tiered decision-making structure, such that decisions can be made by an individual public firm i, by a group of public firms $\vec{\imath} \subset B$ (e.g. a regional group), or by the public employment program as a whole. Decisions of different types are assigned to different program levels. Disputes not resolvable at the firm level are settled by arbitration bodies that aim for unbiased application of the program's governing rules.

For some types of decisions, it may be appropriate to use majority rule voting, whether in the form of direct voting, voting on representatives, or a combined system.[17] But majority decisions are constrained by the rules of the program, e.g. those ensuring that individual participants l can choose freely among work options, and receive fair compensation according to their contribution. Precise internal decision-making rules are allowed to vary from one public firm to another, in such a way as to allow an evolutionary process that incorporates trial and error, and foot voting by program participants.

Per-worker public employment subsidies $\sigma_l^{j \in B}$ are determined by formulae similar to those commonly used to determine private sector subsidies $\sigma_l^{i \in A}$, which include unemployment subsidies σ_l^0. That is, they may depend on current income, income history, assets, number and age of dependents, etc.

Profit shares are divided evenly among program participants, either at the firm level, the program level, or some combination of the two. When profit division takes place at the firm level, the number of workers $|L \cap x_i|$ at a single firm $i \in B$ may be decided outside the firm to avoid pressure from existing workers against new hires.

Workers in the public employment program are given maximal control over their own working environments. For example, a worker wishing to spend fewer weekly hours on the job is allowed to do so without penalty, provided that he is willing and able to take a pay cut

equivalent to the resulting loss in output, or the cost of hiring additional labor to maintain the same output.[18] Further, the principle that all workers must be treated with respect, regardless of the tasks they perform, is a programmatic directive of the first order, subordinate to no other concern.

In some cases, it may be advantageous to pay workers (as well as capital units and land units) in shares of their product's value when finished. For a simple example, suppose that a public firm $i \in B$ produces a single product with a value $y_i(x_i)$ that is not perfectly known during production, but which will be revealed upon completion. Each contributing input unit $z \in x_i$ would receive a share s_z^i such that $\sum_{z \in x_i}(s_z^i) = 1$. A person receiving such a share could either liquidate it immediately on an open spot market in exchange for a payment of $s_z^i \cdot \hat{y}_i(x_i)$, where $\hat{y}_i(x_i)$ is determined by the market at that time (and discounted to present value), or hold the share until it realizes its final value $s_z^i \cdot y_i(x_i)$.

§ 7.3. Connecting discussion

In implementing a per-worker subsidy scheme that mirrors the private sector, the public employment program attempts to minimize the number of workers l such that $\Delta_l^{\alpha_l} + v_l^{\alpha_l} > \Delta_l^{\beta_l} + v_l^{\beta_l}$ but $\Delta_l^{\alpha_l} + v_l^{\alpha_l} + \sigma_l^{\alpha_l} < \Delta_l^{\beta_l} + v_l^{\beta_l} + \sigma_l^{\beta_l}$. It is likely that some such individuals will still exist, but any resulting efficiency loss should be considered in combination with the program's ability to reduce the need for subsidies in both sectors.

The possibility of increased transaction cost in productive decision-making is a natural concern with various proposals for workplace democratization. One can argue that stockholders in a (simplified) private firm are able to easily reach unified decisions because they have the unified goal of maximizing profit. In contrast, workers in a participatory firm may have diverging goals, e.g. when it comes to how tasks are divided. No single solution to this concern is obvious; therefore we suggest a pluralistic system that can evolve with trial, error, and foot voting.

In holding administrators accountable for projects that produce negative surplus, and in rendering some payments to inputs w_z^i in the form of output shares s_z^i, the public employment program attempts to minimize the incidence of unexpected losses, $\Pi_{i \in B} < 0$. The former provision aims to put decision-making power regarding uncertain investments in the hands of those who have demonstrated investing competence. The latter provision makes use of an open spot market to inform predictions of investments' future values, to spread the risk associated with investment failure, and (to the extent that workers hold their shares) to provide added incentives for doing good work.

The policy principles of allowing maximal worker autonomy by default, and emphasizing respectful treatment of all workers regardless of station, follow from the aim of creating a production sector in which the goal of maximizing worker utility is brought to the forefront. This amounts to a shift in emphasis, from a paradigm in which workers sell their time to a firm that hopes to profit from their efforts, to a paradigm in which the firm serves a means by which workers aim to achieve a favorable combination of workplace utility and income-

generating production. In an idealized mathematical model of a firm, this distinction might effectively disappear under conditions such as perfect competition, etc. But in the lived experience of human labor, the shift in emphasis may provide several benefits ranging from obvious to subtle. For example, because real labor has a psychological dimension, job satisfaction is driven by social factors such as respect and esteem, as well as by material compensation. To the extent that much work available to low-skilled labor in the private market is socially humiliating, it creates a friction against labor force participation. By emphasizing worker satisfaction as well as productivity, the public employment program may be able to provide a labor force entry point without this friction.

Regarding the possibility of effective insider groups forming within the public employment program and seeking gains at the expense of other workers, we consider again that there is no single, obvious solution. Just as insider groups form in private firms, they may find other ways to form in various types of socialist environments. The question then is whether it is possible to reduce the frequency and incidence of their formation. As in the private sector, a worker marginalized by an insider group (which e.g. knowingly pays him less than his contribution Δ_i^i) can quit, and join either a private firm or another public one. In addition to this standard exit option, the public employment program provides the option of filing a grievance and initiating the arbitration process. No such process can be perfect, but a reasonably well-functioning version of this could provide an advantage relative to the private sector, where firms have no obligation whatsoever to pay workers according to their contribution as opposed to their reservation wage, or to give them a share of the profits.

§ 8. Conclusion

In this paper, we have introduced an idea for a public employment program. The program is organized as a federation of quasi-independent public firms that produce goods and services, employ capital and land at their respective opportunity costs, and divide the remaining revenue from production among workers according to their respective contributions.

The motivation of the program is to create a labor market in which workers can find employment where they are treated as insiders rather than outsiders, i.e. ends rather than means.

Our model flows logically from the program's motivation. That is, we explore an environment in which input units and firms are heterogeneous because we are concerned with cases where profits Π_i can be positive, reservation wages $\underline{w}_z^i$ can diverge from contributions Δ_z^i, and thus an input unit z may receive different compensation depending on whether it is an insider $z \in \mathrm{M}_i$ or an outsider $z \in \mathrm{N}_i$.

Further, we allow for the existence incomplete information (modeled as a restriction of available firm-input configurations, $\xi \in \Omega \subsetneq \Xi$) because we are concerned with cases where potential gains from productive exchange go unrealized, leading to involuntary

unemployment, and other forms of suboptimal employment. In these cases the program endeavors to add social value by coordinating productive opportunities on the behalf of those workers who are most adversely affected by this kind of labor market failure.

It is possible to consider this program in an environment without any subsidies at all. However, if prior personal subsidies $\sigma_l^{\alpha_l}$ exist in the form of social assistance payments, efficient allocation of workers between the two sectors requires the public employment program to match these subsidies, $\sigma_l^{\beta_l} = \sigma_l^{\alpha_l}$. Since workers receive these subsidies *in addition to* their compensation from productive activity, even those who can only make relatively small contributions to the production process are able to achieve a higher standard of living than they could if unemployed. Further, we highlight one particular type of subsidy scheme that accomplishes redistribution without a loss in efficiency: a lump-sum grant financed by a tax on unimproved land and other raw natural resources.

In the baseline model, we find that the program has a positive effect on efficiency. When we develop successive variations on the baseline model, we reassert this result in some cases, while also introducing important caveats. In the variation where expansion of the information set is costly, the result can be reasserted as long as this administrative cost $\sum_{a\in\Phi}(c_a)$ is accounted for in the program's non-negative profit constraint. In the variation where the program offers greater subsidies to some individuals l than they would receive in the private sector, $\sigma_l^{\beta_l} > \sigma_l^{\alpha_l}$, the result depends on how large these overpayments are relative to the surplus generated by public firms. In the variation where the program can err in assessing individual contributions Δ_z^i, the result can be reasserted because any efficiency loss associated with a misallocation of input unit to the public employment program due to overpayment must be outweighed by other surplus that the program creates. In the variation where the future value $y_i(x_i)$ of current investments is uncertain, the result depends on the program's ability to impose a reasonable level of discipline on investment losses.

Based on the program's goals and potential vulnerabilities, we make tentative suggestions for particular features of its design. These features include partial independence of public firms, incentives for administrators that align their interests with social goals, a tiered decision-making structure, a per-worker subsidy-determination formula that mirrors the private sector, an emphasis on flexible employment conditions and respectful treatment of workers, and an option of paying wages in terms of output shares, where shares can be liquidated via an open spot market before the final products are sold.

These suggestions are not meant to be definitive, and we do not assert that they answer all important program design questions. Rather, they are intended to stimulate the imagination of readers who are willing to consider what mixes of market mechanisms, bureaucratic mechanisms, and democratic mechanisms might be most effective at achieving the program's goals. If others take up this challenge, this question could emerge as a rich and rewarding frontier in social choice theory.

In conclusion, we do not assert that a public employment program like the one described here is a desirable policy under all circumstances. We have identified conditions under which it can generate inefficiency, and acknowledged that many details remain to be worked out.

However, what we do suggest is that the idea has an elegant conceptual appeal, sufficient to warrant further consideration.

That is, because the program proceeds by creating new productive opportunities rather than restricting or taxing existing ones, it is intuitive that it might improve efficiency under a broad range of conditions. Because it prioritizes the creation of opportunities for those least well-served by the private market, and improves the bargaining power of labor relative to capital and land, one can make a clear argument for it the grounds of equity.

The program creates a public production sector, which implies certain organizational challenges. But when favorable conditions such as balanced subsidies and limited incidence of investment failure are met, the program only grows by attracting inputs on a voluntary basis, and thus only when its growth adds value. The result is an efficient voting-with-the-feet equilibrium, in which those inputs best suited for each production sector are employed there.

Endnotes

[1] See e.g. Varian (2014), Chapters 20-21.

[2] This result, known as Euler's Theorem, is one of the oldest chestnuts in economic theory. See e.g. commentary by Robinson (1934) and Samuelson (1952).

[3] Dow (1993) makes the distinction between capital hiring labor and labor hiring capital, and observes that the former is much more common than the latter.

[4] The ability of microcredit programs — as defined e.g. in Yunus (2003) — to increase economic efficiency implies this kind of initial information failure. Thus, there is a philosophical connection between microcredit programs and the public employment programs described here, in that both aim to unleash the potential productivity of workers by allowing them access to complementary production inputs on a more efficient basis.

[5] Cooper and John (1988) provide an early formalization of this idea.

[6] Tiebout (1956) introduces an analysis of foot voting with respect to local public expenditures.

[7] In some ways, this idea of foot voting between public and private options is analogous to the "public option" for health insurance that was included in the "Affordable Health Care for America Act" — or HR 3962, passed by the US House of Representatives in November 2009 — but not included in the "Patient Protection and Affordable Care Act" signed into law in March 2010. See Halpin and Harbage (2010).

[8] As discussed e.g. in Wray (1997) and Wisman (2010).

[9] Thus we suggest a natural complementarity between the type of public employment program described here, and a land tax as advocated by George (1879) and his modern exponents.

[10] Note that a special case with $\sum_{z\in x_i}(\Delta_z^i) > y_i(x_i) > \sum_{z\in x_i}(\underline{w}_z^i)$ is possible, i.e. when a firm i uses inputs $z \in x_i$ that are highly complementary and difficult to replace. In a subcase of this, even a profitable firm i with $y_i(x_i) > \sum_{z\in x_i}(\underline{w}_z^i)$ may not be able to pay its insider units $z \in \mathrm{M}_i$ their full replacement costs Δ_z^i. But we can still assume that such a firm will seek to maximize payments to insider units $z \in \mathrm{M}_i$ while employing useful outsider units z: $\Delta_z^i > \underline{w}_z^i$ at their reservation wages $\underline{w}_z^i$. And we can still hold that such a firm will be able to pay each input z up to Δ_z^i in the presence of sufficiently high competing bids w_z^j — because otherwise the profitability condition $y_i(x_i) > \sum_{z\in x_i}(\underline{w}_z^i)$ would be violated.

[11] Thus, we incorporate in simplified form a result that the literature discusses in more detail; see e.g. Tideman (1982).

[12] As with Proposition 1, it is possible to construct a parallel analysis here which allows for the simultaneous movement of multiple factors. In this analysis we can let $\vec{z} \to \vec{\imath} \to \vec{\jmath}$ indicate that a set of input units $\vec{z}$ moves in tandem from a set of initial employing firms $\vec{\imath}$ to a new set of employing firms $\vec{\jmath}$, and denote the old and new employment assignments for $z \in \vec{z}$ as $\vec{\imath}_z$ and $\vec{\jmath}_z$, respectively. We can further define
$\Lambda^{\vec{\imath}}_{\vec{z}} = \sum_{i \in \vec{\imath}} \Big(\min_{\vec{z}' : \vec{z} \cap \vec{z}' = \emptyset} \{ [y_i(x_i) - y_i(x_i \backslash \vec{z} \cup \vec{z}')] + [\sum_{z' \in \vec{z}'} (\underline{w}^i_{z'})] \} \Big)$ and
$\Lambda^{\vec{\imath}}_{\vec{z}} = \sum_{i \in \vec{\imath}} \Big(\max_{\vec{z}' : \vec{z} \cap \vec{z}' = \emptyset} \{ [y_i(x_i \cup \vec{z} \backslash \vec{z}') - y_i(x_i)] + [\sum_{z' \in \vec{z}'} (\underline{w}^i_{z'})] \} \Big)$ as the multi-input versions of the 'replacement cost' and 'contribution' definitions of Δ^i_z, respectively. By extending the statement $[z \to i \to j] \Rightarrow [\Delta^j_z + v^j_z > \Delta^i_z + v^i_z]$ to the statement $[\vec{z} \to \vec{\imath} \to \vec{\jmath}] \Rightarrow \left[\Lambda^{\vec{\jmath}}_{\vec{z}} + \sum_{z \in \vec{z}} \left(v^{\vec{\jmath}_z}_z\right) > \Lambda^{\vec{\imath}}_{\vec{z}} + \sum_{z \in \vec{z}} \left(v^{\vec{\imath}_z}_z\right)\right]$, we can substitute these definitions for their single-unit counterparts in Proposition 5 and reach an equivalent result via $[\vec{z} \to \vec{\imath} \to \vec{\jmath}] \Rightarrow \left[\left(\Lambda^{\vec{\jmath}}_{\vec{z}} - \Lambda^{\vec{\imath}}_{\vec{z}}\right) + \sum_{z \in \vec{z}} \left(v^{\vec{\jmath}_z}_z - v^{\vec{\imath}_z}_z\right) > 0\right]$.

[13] To avoid too great a split with our main approach, which is grounded in microeconomics, we do not attempt to model macroeconomic effects here. However, combining elements of this model with elements of existing macroeconomic models seems like a promising area for future work. For example, if the public employment program reduces the economic uncertainty felt by households, this could increase economic activity in the private market. We could model this as an expansion of the information set Ω, brought about by a partial resolution of coordination problems. Adding such features to the model may strengthens the argument for a version of the program that is large enough to guarantee full employment.

[14] To illustrate this point with a simplified example, suppose that $\sigma_l^{\alpha_l}$ and $\sigma_l^{\beta_l}$ are two random variables with the same distribution. In this case the statement $\Delta_l^{\alpha_l} + v_l^{\alpha_l} + \sigma_l^{\alpha_l} < \Delta_l^{\beta_l} + v_l^{\beta_l} + \sigma_l^{\beta_l}$ is more likely to be false than true given the statement $\Delta_l^{\alpha_l} + v_l^{\alpha_l} > \Delta_l^{\beta_l} + v_l^{\beta_l}$, but the statement $\Delta_z^{\beta_z} + v_z^{\beta_z} + \sigma_z^{\beta_z} > \Delta_z^{\alpha_z} + v_z^{\alpha_z} + \sigma_z^{\alpha_z}$ is more likely to be true than false given the statement $\Delta_z^{\beta_z} + v_z^{\beta_z} > \Delta_z^{\alpha_z} + v_z^{\alpha_z}$.

[15] In cases where workers' contributions are exceedingly difficult for public firms to assess, one possible response is to flatten out their wages overall, thus making the program more similar to a standard ELR proposal.

[16] Albert and Hahnel (1991) also take up the challenge of designing socialist productive institutions, though their approach differs from ours in several ways. For example, they imagine socialism replacing capitalism, rather than coexisting with it as an alternative that workers may take advantage of on an individual basis. Also, they prefer to reward workers in proportion to their apparent effort rather than according to the value that they add to production.

[17] Miller (1969) and Green-Armytage (2015) develop systems in which, on each issue, each voter can either vote directly or delegate his vote to a proxy of his own choosing.

[18] In theory, this is a Pareto-improving change, but it does not always occur in private employment, in which some workers remain 'overemployed', working more hours than they want to. Sousa-Poza and Henneberger (2002) review some overemployment statistics; they report that that the percentage of workers who would prefer to work less even if it meant getting less pay is on the order of 10%, with the exact number varying by country and year. Further overemployment data is available via the May 1985 Current Population Survey, the 1992, 1997 and 2005 Work Orientations Survey given as part of the International Social Survey Programme, and the Panel Study of Income Dynamics from 1968-1987.

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